Reasoning about Collective Action in Markov Logic: A Case Study from Classical Athens

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Abstract. Solving the collective action problem is to understand how people decide to act together for the common good when individual rationality would lead to non-cooperative selfish behaviour. Two important features that can foster collective action are achieving common knowledge about the problem faced and the existence of a shared cooperative ethos. Based on the work of Ober, who argued that the success of classical Athens was the result of its shared commitments, social values and specific procedural rules, we define a probabilistic model in Markov Logic of a specific prosecution against an Athenian trader who neglected to contribute to the city when it was in a crisis. In order to join together for a common good, our model focuses on a decision-making approach based on the aspects of common knowledge. In particular, the reasoning agent will be able to make a decision based on the predicted cooperation level of citizens given the result of the trial. We expect that our computational model of this case study can be generalised to other problems of reasoning about collective action based on common knowledge in future work.

Keywords: Common Knowledge. Collective action · Markov Logic Networks

1 Introduction

Solving the collective action problem is to understand how people act together for the common good. Collectively reducing the emission of greenhouse gases [20] and managing common pool resources [18] are two examples of environmental collective actions. Solving collective action problems is important to overcome various social and environmental problems. A collective action requires communication, organisation, and incentives that motivate everyone to work together for the common good. Existing studies [18, 19] introduce strategies and solutions to tackle these problems. Holzinger [10] discusses various solutions including norms, rules and sanctions. Prior work in the field of game theory [15] proposed solution concepts such as the Nash equilibrium [9]. However, the reasoning for this gets complicated for a large number of agents, and it may not be an effective system for human reasoning. Besides common knowledge, expectations and credible commitment also motivates people to join together for collective action.

Political scientist Josiah Ober [17] discusses the role of common knowledge in making people join collective actions in classical Athens. He argues that Athens was socially, politically and militarily successful compared to rival states because of a superior ability to achieve shared commitments, shared social values and procedural rules through common knowledge. In particular, a specific trial discussed by him has a lot of richness and information regarding the cooperation and social structure in terms of common knowledge. Therefore, we adopt this as a case study of the role of the common knowledge in achieving collective action.

The trial was against an Athenian trader named *Leocrates*. It was alleged that he abandoned the city when it needed help to defend, and reconstruct it after a battle had been lost with the Macedonians. Lycurgus, a famous Athenian politician, prosecuted this trial with an intention of convincing the jurors to convict *Leocrates* for the capital charge of treason. A record exists of the narrative Lycurgus gave to persuade jurors of the importance of convicting *Leocrates*.

The trial is an interesting case study in which we found the prosecutor's points convey the importance of common knowledge in fostering the collective goal of having a secure city and how the result of this trial will impact the security of the city. Therefore, we are interested in implementing a computational model using common knowledge to find how agents would make decisions based on a logical encoding of some of the arguments made by the prosecutor.

We consider several sources that lead to attaining common knowledge: the community acceptance of a collective goal through observation of an alignment cascade [21], measures of common knowledge about social attitudes to the collective goal through empirical observations, and observing states of affairs that satisfy four conditions identified by Lewis [14] as giving rise to common knowledge.

As Markov logic networks (MLNs) [5] express knowledge explicitly, and also help representing beliefs of a probabilistic nature, we use them to model this trial. For example, a belief that a certain proportion of citizens are cooperative with city-wide goals is probabilistic in nature, based on an estimate of the percentage of cooperators.

The paper is structured as follows: The concept of common knowledge is discussed in Section 2, along with how it will be helpful to achieve a common goal. The discussion in Section 3 centers on points in terms of common knowledge made by Lycurgus during the trial of *Leocrates*. As the trial is modelled using a Markov Logic Network (MLN), Section 4 provides an overview of MLNs. A description of how this trial was modelled as an MLN is provided in detail in Section 5. Section 6 discusses how the model can be queried to inform the decision of a juror in the trial. Section 7 concludes the paper.

2 Common Knowledge

A range of studies discuss the importance of knowledge alignment in bringing people together. In fact, knowledge and action are intimately connected. In most situations, people act according to what they think. The term 'common knowledge' refers "knowledge of what other people know about other people's knowledge" [4]. According to Kuhlman et al., [13] "Successful coordination requires that people know each others' willingness to participate, and that this information is common knowledge among a sufficient number of people." This involves infinite information transmission levels which can be explained [14] as:

- I know something; you know something
- I know that you know; you know that I know
- I know that you know that I know
- You know that I know that you know; and so on ...

Social coordination revolves around the achievement of common knowledge. From knowing where to find your partners, to communicating with them, to resolving public goods dilemmas, to following social norms, success in social interactions often depends on common knowledge. Achieving common knowledge requires collective awareness and collective attention.

Ober [17] discusses how common knowledge was used in classical Athens to collect people for a common goal. Public rituals for honouring war heroes and monuments containing the list of traitors were a medium for spreading common knowledge that every citizen should act for the good of the city. Athenians used certain specific signs in temples to convey important messages to their citizens. Festivals were organized so that every citizen was forced to pass through them. The assumption was that if the signs were placed in public places, everyone will be able to see them.

In Athens, People's Courts sat frequently, and the relatively long speeches of litigants provided excellent opportunities for sharing knowledge. Court was one of the places where collective knowledge was developed and used. The jury was drawn from the same citizens. Citizens attended the court to observe the jurors' presentations during which they observed the responses of others, like facial expressions and exclamations. Prosecution points from the *Leocrates* trial argue for the importance of reaching collective action in terms of common knowledge.

Lewis [14] provided a game theoretic solution for coordination problems which considers the relation between common knowledge and mutual expectation. In explaining a choice of action, he says that the agent needs a reason to believe about what actions will be chosen by others. Then an equilibrium is sustained due to mutual expectations which comes from common knowledge.

There are some properties that allow us to know when it is appropriate to recognize common knowledge based on a certain principles. Lewis separates the concepts of directly observable states of affairs and how they provide reason to believe certain proposition: this is modelled as an "indicates" relationship (a state of affairs indicates that a proposition holds). Lewis identified four properties that allow a state of affairs (s) to be recognised as a "reflexive common indicator" of a proposition (p), i.e. that observation of the state of affairs leads to common knowledge of the proposition. The state of affairs should be self revealing and public, everyone should be able to infer p from observing s and every one should have reason to believe they share the same inductive standards' and background information.

In the context of the trial that we model, Lewis's theory explains how observing information of past traitors and their convictions can be reached without explicit logical reasoning about infinitely nested knowledge operators. In addition, we consider another source of common knowledge noted by Ober: a cascade of actions by citizens to help rebuild the city's walls at a time of crisis, in response to a public decree to act collectively secure the city.

3 Points of Lycurgus During The Trial

During his time in Athens, Lycurgus, the prosecutor of the trial was a famous politician who performed many social services. He had successful past prosecutions of citizens who acted against security rules. During the trial of *Leocrates*, who was alleged to have abandoned the city, Lycurgus's speech focused on two main equilibria. The first one is a shared belief that is common knowledge among the citizens that everyone should cooperate to secure the city. As Athens' security was viewed as a common pool resource every citizen should play their part in ensuring it. Unless individuals give back to the common pool, it leads to the tragedy of the commons [8].

The second equilibrium is that jurors should penalize citizens who violate the first equilibrium through legal sanctions. As the city had been completely destroyed and lost territory after the war with the Macedonians. Athenian's cooperation was crucial during the period when *Leocrates* was alleged to have abandoned it. Other citizens of Athens committed themselves to rebuilding the city. For the common good to be achieved, there was a need for cooperation among citizens at the time. However, it was alleged that *Leocrates* had left the city by disobeying the generals and ignoring the shared interest of citizens of the city.

The above points are taken from Lycurgus's preserved speech [3, 1]. We are interested in reconstructing the key arguments of Lycurgus as a case study in computational reasoning about common knowledge and collective action in terms of MLNs. In particular, the focus of our model is on the reasoning that must be performed by the jurors. They need to understand the effect that their decision about conviction will have on the maintenance of the citizens' cooperation with the group goal to keep the city secure.

4 Markov Logic Networks (MLNs)

Two important aspects of artificial intelligence (AI), expressing knowledge and uncertainty, can be handled with first-order logic (FOL) and probability, respectively. These both can be learned or combined with inference mechanisms. There are various approaches to combining probabilistic reasoning with explicit logical knowledge encoding such as probabilistic relational models [7], Bayesian logic programs [12], relational dependency networks [16].

Richardson and Domingos [5] proposed a logic framework Markov Logic Networks (MLNs). While a first-order logic (FOL) knowledge base contains formulas that can be seen as hard constraints on the possible worlds (assignments of truth values to ground atoms), in an MLN, each formula has an associated weight that reflects how strong a constraint it is. The higher the weight, the greater the difference in log probability between a world that satisfies the formula and one that does not, other things being equal. That allows to model the probabilistic nature of the formulas.

An MLN can be seen as a template to generate a Markov network (a type of undirected graphical model), given a finite set of constants, and this can be used to answer queries about the conditional and unconditional probabilities of specified ground formulas. An MLN contains a node for each possible ground atom, and has undirected edges connecting nodes that appear together in at least one grounding of a formula in the

MLN. While MLNs are built from FOL formulas, inference is performed using the generated Markov network.

Formally, an MLN is given by a set of pairs (F_i, w_i), where F_i is a formula in first-order logic and w_i is a real-valued weight. Given a finite domain of discourse a set of constants the ground Markov network generates a probability distribution over the set of possible worlds χ as follows,

$$P(\chi = x) = \frac{1}{Z} \exp\left(\sum_{i=1}^{|L|} w_i n_i(x)\right) \quad (1)$$

$Z = \sum_{x' \in \chi} exp(\sum_i w_i n_i(x^{'}))$ is a normalisation constant and $n_i(x)$ denotes the number of groundings of F_i that are true in x.

Given a formula F, abbreviating the presentation of Jain [6], we define:

$$s(F) = \sum_{x \in \chi,\, x \models F} \exp\left(\sum_{i=1}^{|L|} w_i n_i(x)\right) \quad (2)$$

The outer sum is over possible worlds in which F is true and the exponentiated inner sum is the unnormalized probability of the possible world x. Using $s(F)$ we can calculate the probability of any ground formula F_1 given any other ground formula F_2 as

$$P(F_1|F_2) = \frac{s(F_1 \wedge F_2)}{s(F_2)} \quad (3)$$

5 An MLN Model of Lycurgus's argument

In this section we present an MLN model of key aspects of Lycurgus's prosecution speech, which is shown in Listing 1. The model shows how a jury can decide whether to prosecute *Leocrates* using the following two conditional probability queries $Q1$ and $Q2$. These ask what is the likelihood of random citizen *Polites* cooperating with a collective goal to secure the city when *Leocrates* is convicted and not convicted, respectively:

$$Q1: \; P(\mathit{cooperate}(\mathit{Polites}, \mathit{SecureCity}) \mid \mathit{convicted}(\mathit{Leocrates}))$$
$$Q2: P(\mathit{cooperate}(\mathit{Polites}, \mathit{SecureCity}) \mid \neg\, \mathit{convicted}(\mathit{Leocrates}))$$

There are two levels of collective action in this scenario: the citizens securing the city as a collective goal, and the jurors collectively agreeing to convict *Leocrates*. Our model currently includes only the first level. Lycurgus believes that there are two equilibrium conditions. Everyone should strive to secure the city as a common objective. That is the first equilibrium. The second equilibrium is for those who violate the first one to be punished. The jurors are responsible for maintaining the court's ethos of convicting traitors, thereby maintaining the equilibrium of people cooperating towards a common goal. It is specifically our concern that each juror can understand the prosecutor's

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// Domain declarations
dom_citizen = {Polites, Leocrates, Hipparchus, Callistrus}
dom_institution = {Court}
dom_individual_ethos = {Ethos1, Ethos2}
dom_institutional_ethos = {Ethos3}
group = {Citizens}
goal = {SecureCity}

//Predicate declarations
individual_ethos(dom_citizen, dom_individual_ethos!)
institutional_ethos(dom_institution, dom_institutional_ethos!)
ck(group, goal)
group_goal(group, goal)
cooperate(dom_citizen, goal)
convicted(dom_citizen)
traitor(dom_citizen)
prosecuted(dom_citizen)
historic(dom_citizen)

//Background knowledge
ck(Citizens, group_goal(Citizens, SecureCity)).
ck(group, group_goal(group, goal)) =>
    group_goal(group, goal).

cooperate(x, SecureCity) ^ !historic(x) =>
    individual_ethos(x, Ethos1) v individual_ethos(x, Ethos2).

log(0.12) individual_ethos(Polites , Ethos1) ^
    cooperate(Polites , SecureCity)
log(0.48) individual_ethos(Polites , Ethos2) ^
    cooperate(Polites , SecureCity)

traitor(x) <=> ((EXIST g (group_goal(Citizens, g) ^
    !cooperate(x,g)))).

//Definitions
individual_ethos(x, Ethos1) <=>
    (!historic(x) ^
    commonKnowlege(Citizens, group_goal(Citizen, SecureCity)) =>
    cooperate(x, SecureCity)).

individual_ethos(x, Ethos2) <=>
    (!historic(x) ^
    commonKnowlege(Citizens, group_goal(Citizen, SecureCity)) ^
    institutional_ethos(Court, Ethos3) =>
    cooperate(x, SecureCity)).

institutional_ethos(Court, Ethos3) <=>
    (!(EXIST x (traitor(x) ^
    prosecuted(x) ^ !convicted(x)))).
```

Listing 1: MLN encoding of Lycurgus's arguments

arguments and realise that "if I convict *Leocrates*, I'm enhancing the collaboration among citizens otherwise, I'm undermining it".[3]

There is also a set of ground atoms accompanying Listing 1 representing firm knowledge about the domain. It states that the named citizens (other than *Polites*) are historic traitors. Among them, two (*Hipparchus* and *Callistrus*) of them were prosecuted and convicted while one (*Leocrates*) is prosecuted and waiting for the jurors' decision. To evaluate $Q1$, *Leocrates* is declared convicted, for $Q2$ he is not convicted. The details of traitors in these ground atoms can be found on monuments placed in public places where the list of traitors are carved and hence these ground atoms are common knowledge.

Listing 1 shows the structure of an MLN that represents the scenario, implemented using ProbCog[4]. The listing uses nested function symbols, to represent a group goal as a complex term within common knowledge modality. However, since ProbCog does not handle terms with nested function symbols our MLN uses a standard transformation [2] to eliminate these functional terms. For brevity we do not show the transformed version here.

Listing 1 starts with domain declarations that allow a set of constants to be associated with a named domain. Next, every predicate in the MLN is declared. A predicate declaration consists of the predicate name followed by a comma-separated list of domain names of its arguments in brackets.

We consider there to be a prototypical citizen named *Polites*. The Greek word *Polites* refers to a general citizen in Athens [24]. Our aim is to infer the probability of an arbitrary current citizen (*Polites*) cooperating with the shared goal to secure the city, without the known behaviour of a few past defectors having an undue influence on this inference. Given that *Polites* is a single constant representing a large number of citizens of Athens[5], the past traitors (including Leocrates, who is being prosecuted in absentia some time after leaving the city), are modelled as "historic" and the MLN clauses defining the current citizens' ethoses regarding cooperation with the goal explicitly exclude consideration of historic citizens.

Polites is observing the trial and he has believes there is common knowledge of the existence of a group goal of the citizens to secure the safety of the city (e.g. by strengthening its defences). This is shown in the line 22 of Listing 1. Lycurgus argues that this common knowledge comes about from a cascade [21] of action in cooperation with this goal when the city was in danger. Lines 23 to 24 express a deduction that can be made from common knowledge. It declares "When there is common knowledge that a group has a goal, then the group has a goal".

The reasoning agent (a juror listening to Lycurgus's argument) needs to understand how the group goal affects the actions of the citizens. According to Tuomela [23], when members of a group are acting collectively in "we-mode", "one adopts the group's

[3] We do not attempt to model any reasoning about whether a citizen prosecuted for treachery really is guilty. Instead we focus on the argument for conviction (assuming guilt) based on the upholding of social order. In fact, in the real scenario, *Leocrates* was not convicted, as evidence of his guilt was not convincing to the court.

[4] https://github.com/opcode81/ProbCog

[5] MLN inference does not scale well [22], so explicitly modelling a large number of citizens is not feasible.

constitutive goals, values, norms, and standards—briefly its 'ethos' ".[6] We assume that citizens may follow one of two possible ethoses in regards to securing the city (line 11 and lines 26 to 27). The ! indicates a functional relationship: the indicated argument is uniquely defined given the other arguments of the predicate. These annotations in lines 11 and 12 means that an individual or institution can have at most one ethos.

Polites observes these two competing individual ethoses that citizens have regarding cooperation with the goal to secure the city. Ethos 1 (lines 38 to 41) is to unconditionally cooperate with a group goal. The Ethos 2 (lines 43 to 47) is more selfishly to cooperate only if the court holds the ethos of convicting traitors.

Moreover, empirical knowledge about the proportions of cooperating agents in the city, and the proportions of agents holding Ethos 1 and 2 amongst those agents[7] is encoded using weights on the mutually exclusive joint probabilities in lines 29 to 32. *Polites* is capable of estimating the proportion of citizens who cooperate with the group goal, and who hold each ethos based on the background knowledge of current status of cooperation of the city. The background knowledge comes from the observation of public interactions (building walls, public oath), shared cultural information (honoring heroic warriors, celebrating war victories) and the present shared situation which all are matter of common knowledge. We assume that *Polites* has observed 60% cooperation with the group goal, a 20% incidence of Ethos 1 amongst cooperators, and an 80% holding of Ethos 2 amongst them.

Ethos 3 is defined as "No traitor who is prosecuted will not be convicted" (lines 49 to 51)[8]. The weight of Ethos 3 is determined empirically based on historic common knowledge, which is expressed in the set of ground atoms. *Polites* knows about some past traitors (*Hipparchus*, *Callistrus*) who were prosecuted, and the outcomes of the trials. He can infer empirically how strongly the court has the ethos of convicting or acquitting traitors based on those decisions.

Lines 34, 35 define the term traitor: "A traitor is someone who doesn't cooperate with the group goal". This is also common knowledge which comes from the cultural background and shared knowledge that everyone knows what a traitor is.

All the clauses in Listing 1 come either from common knowledge, or are assumed to be common knowledge e.g., the observed proportion of cooperation and prevalence of Ethos 1 and 2. Some clauses describe common knowledge explicitly. In other cases, the rules themselves are common knowledge. The reasoning of an agent is making an assumption that other agents also have similar observations. This is also common knowledge. As this MLN was constructed exclusively from common knowledge then the conclusions of reasoning with it are also common knowledge. If a juror believes that all this knowledge is common, then the conclusions reached by the MLN queries can also be considered common knowledge amongst the jurors, thus encouraging a consensus decision to convict or not.

[6] According to Toumela [23] there is a mutual belief that, if a group has set of ethoses, all its members are collectively committed and accepted to that ethoses. Essentially this is common knowledge.

[7] Due to difficulties in expressing conditional probabilities in MLN clauses [11], this knowledge is expressed in terms of joint probabilities of cooperation and holding a certain ethos.

[8] ProbCog provides an "exist" operator but not a "for all" one.

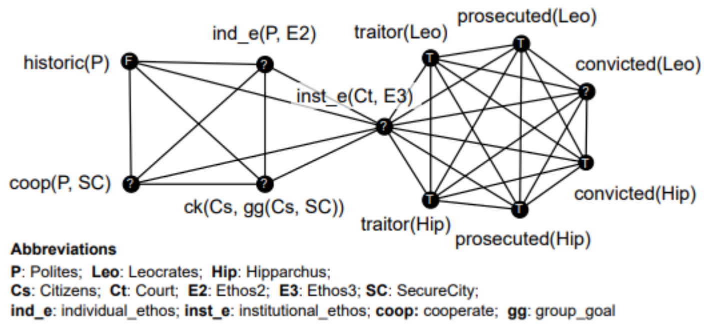
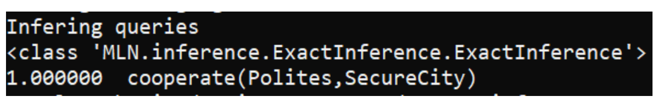


Fig. 1: Part of the Markov Network generated from Listing 1

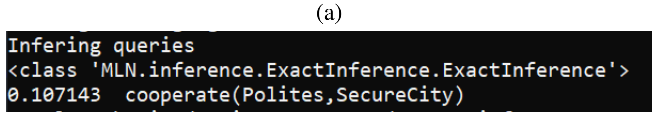
Figure 1 shows an excerpt from the Markov network generated from the MLN and the chosen set of constants. The left hand side shows a grounding of the clause defining Ethos 2 (where x = Polites), while the right hand side is a partial depiction of the single grounding for the clause defining Ethos 3 (for brevity, only citizens *Leocrates* and *Hipparchus* are considered here). The nodes are annotated with F or T where their truth values are fixed by the set of ground atoms, and ? where the truth value is not fixed and may vary between possible worlds. The figure illustrates how the probability of Polites cooperating with the goal to secure the city is influenced by the probabilities of various other ground atoms.

6 Results and Discussion



```
Infering queries
<class 'MLN.inference.ExactInference.ExactInference'>
1.000000  cooperate(Polites,SecureCity)
```

(a)



```
Infering queries
<class 'MLN.inference.ExactInference.ExactInference'>
0.107143  cooperate(Polites,SecureCity)
```

(b)

Fig. 2: Results obtained from probcog tool to compute queries $Q1$ and $Q2$ respectively

We are interested in finding the likelihood of *Polites* and thus the average citizen cooperating due to the common knowledge he/she received from the observation of the city. In particular, what is the probability that *Polites* will cooperate when *Leocrates* is convicted and when he is not? Using conditional probability queries, we can draw some conclusions as common knowledge, since all the input is associated with common knowledge. The result of convicting *Leocrates*, will be to reinforce knowledge of the court's ethos of punishing traitors, and thus to increase cooperation by Ethos2 holds. This will motivate a jury member to vote to convict *Leocrates* to uphold cooperation with the citizens' goal to keep the city secure.

Using the ProbCog tool with the exact inference mechanism, we obtained results of a random citizen's probability of cooperation (cooperate(*Polites*, *SecureCity*)) in both situations of conviction and non conviction of *Leocrates* (Figure 2a and Figure 2b respectively.When $Q1$ and $Q2$ are computed using Equation 3, in the case of the court convicting *Leocrates*, *Polites* will cooperate with probability 1.00 and 0.12 for when he is not convicted. The truth value for convicted (*Leocrates*) is defined differently for these two queries.

Given these predictions, a jury member can validate the argument of Lycurgus using this reasoning and as the conclusion comes from common knowledge he can be confident his opinion will align with that of other jurors.

7 Conclusion and Future work

People act collectively for various reasons, and we are interested in knowing what makes them act as a group. Common knowledge plays an important role in bringing people together at the social level. We provided a computational model to show how cooperation will be achieved on the basis of common knowledge by investigating a specific trial of classical Athens. We used an Markov Logic Network (MLN) as it is capable of combining logical and probabilistic reasoning. Based on Lycurgus's argument we assume that the clauses in our MLN are common knowledge including ethoses, background knowledge about the term traitor, and proportion of cooperation. The reasoning agent observes these and it has a reason to believe it is common knowledge.

Our future work will focus on building a simulation of this scenario in which common knowledge is created and assembled to form the MLN presented in this paper. This will happen in three scenes (1) extract common knowledge of the existence of the group goal from a public decree, followed by an observed cascade of action in cooperation with that goal. (2) observations of information of historic convictions of traitors on public monuments, which, due to a shared cultural and educational background can be seen as reflexive common indicators in Lewis's theory. (3) empirical observations of the ethoses of the citizens towards the group goal. We will use notions such as salience and counts-as relationships between concrete and institutional events to determine which simulated events are candidates for common knowledge. Combining this simulation with MLN reasoning will allow us to show how Lycurgus's complex arguments about common knowledge an joint action can be realised in a computational agent.

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