Reasoning about Opportunistic Propensity in Multi-agent Systems *

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Abstract. Opportunism is a behavior that takes advantage of knowledge asymmetry and results in promoting agents' own value and demoting others' value. We want to eliminate such selfish behavior in multi-agent systems, as it has undesirable results for the participating agents. In order for monitoring and constraint mechanism to be put in place, it is needed to know in which context agents will or are likely to perform opportunistic behavior. In this paper, we develop a framework to reason about agents' opportunistic propensity. Opportunistic propensity refers to the potential for an agent to perform opportunistic behavior. In particular, agents in the system are assumed to have their own value systems and knowledge. With value systems, we define agents' state preferences. Based on their value systems and incomplete knowledge about the state, they choose one of their rational alternatives, which might be opportunistic behavior. We then characterize the situation where agents will perform opportunistic behavior and the contexts where opportunism is impossible to occur.

Keywords: Opportunism, Propensity, Logic, Reasoning, Decision Theory

1 Introduction

Let us first consider an example scenario. A seller sells a cup to a buyer and it is known by the seller beforehand that the cup is actually broken. The buyer buys the cup without knowing it is broken. The seller exploits the knowledge asymmetry about the transaction to achieve his own gain at the expense of the buyer. Such behavior which is intentionally performed by the seller was named opportunistic behavior (or opportunism) by economist Williamson [13]. Opportunism is a selfish behavior that takes advantage of relevant knowledge asymmetry and which results in promoting one's own value and demoting others' value [6]. In the context of multi-agent systems, it is normal that knowledge is distributed among participating agents in the system, which creates the ability for the agents to

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behave opportunistically. We want to constrain such a selfish behavior, as it has undesirable results for other agents in the system. Evidently, not every agent is likely to be opportunistic. In social science, ever since the theory about opportunism was proposed by Williamson in economics, it has gained a large amount of criticism due to over-assuming that all economic players are opportunistic. [4] highlights the challenge on how to predict opportunism *ex ante* and introduces a cultural perspective to better specify the assumptions of opportunism. In multi-agent systems, we also need to investigate the interesting issues about opportunistic propensity so that the appropriate amount of monitoring [7] and constraint mechanisms can be put in place.

Based on decision theory, an agent's decision on what to do depends on the agent's ability and preferences. If we apply it to opportunistic behavior, an agent will perform opportunistic behavior when he can do it and he prefers doing it. Those are the two issues that we consider in this paper without discussing any normative issues. Based on this assumption, we develop a model of transition systems in which agents are assumed to have their own knowledge and value systems, which are related to the ability, and the desire, respectively, of being opportunistic. Our framework can be used to predict and specify when an agent will perform opportunistic behavior, such as which kinds of agents are likely to perform opportunistic behavior and under what circumstances. A monitoring mechanism for opportunism benefits from this result as monitoring devices may be set up in the occasions where opportunism will potentially occur. We can also design constraint mechanisms for opportunism based on the understanding of how agents decide to behave opportunistically. Besides, our framework can be used by autonomous agents to decide whether to participate in the system, as their actions might potentially be regarded as opportunistic behavior given their knowledge and value systems.

In this paper, we introduce a framework to reason about agents' opportunistic propensity. Opportunistic propensity refers to the potential for an agent to perform opportunistic behavior. More precisely, agents in the system are assumed to have their own value systems and knowledge. We specify an agent's value system as a strict total order over a set of values, which are encoded within our logical language. Using value systems, we define agents' state preferences. Moreover, agents have partial knowledge about the true state where they are residing. Based on their value systems and incomplete knowledge, they choose one of their rational alternatives, which might be opportunistic. We thus provide a natural bridge between logical reasoning and decision making, which is used for reasoning about opportunistic propensity. We then characterize the situation where agents will perform opportunistic behavior and the contexts where opportunism is impossible to happen.

2 Framework

We use Kripke structures as our basic semantic models of multi-agent systems. A Kripke structure is a directed graph whose nodes represent the possible states of the system and whose edges represent accessibility relations. Within those edges, equivalence relation $\mathcal{K}(\cdot) \subseteq S \times S$ represents agents' epistemic relation, while relation $\mathcal{R} \subseteq S \times Act \times S$ captures the possible transitions of the system that are caused by agents' actions. We use s_0 to denote the initial state of the system. It is important to note that, because in this paper we only consider opportunistic behavior as an action performed by a hypothetical agent, we do not model concurrent actions labeled with agents so that every possible transition of the system is caused by an action instead of joint actions (see e.g., [2] [11] for related models). For simplification, we assume that the actions in our model are deterministic. We use $\Phi = \{p, q, ...\}$ of atomic propositional variables to express the properties of states S. A valuation function π maps each state to a set of properties that hold in the corresponding state. Formally,

Definition 2.1. *Let $\Phi = \{p, q, ...\}$ be a finite set of atomic propositional variables. A Kripke structure over Φ is a tuple $\mathcal{T} = (Agt, S, Act, \pi, \mathcal{K}, \mathcal{R}, s_0)$ where e.g*

- *$Agt = \{1, ..., n\}$ is a finite set of agents;*
- *S is a finite set of states;*
- *Act is a finite set of actions;*
- *$\pi : S \to \mathcal{P}(\Phi)$ is a valuation function mapping a state to a set of propositions that are considered to hold in that state;*
- *$\mathcal{K} : Agt \to 2^{S \times S}$ is a function mapping an agent in Agt to a reflexive, transitive and symmetric binary relation between states; that is, given an agent i, for all $s \in S$ we have $s\mathcal{K}(i)s$; for all $s, t, u \in S$ $s\mathcal{K}(i)t$ and $t\mathcal{K}(i)u$ imply that $s\mathcal{K}(i)u$; and for all $s, t \in S$ $s\mathcal{K}(i)t$ implies $t\mathcal{K}(i)s$; $s\mathcal{K}(i)s'$ is interpreted as state s' is epistemically accessible from state s for agent i. For convenience, we use $\mathcal{K}(i, s) = \{s' \mid s\mathcal{K}(i)s'\}$ to denote the set of epistemically accessible states from state s;*
- *$\mathcal{R} \subseteq S \times Act \times S$ is a relation between states with actions, which we refer to as the transition relation labeled with an action; we require that for all $s \in S$ there exists an action $a \in Act$ and one state $s' \in S$ such that $(s, a, s') \in \mathcal{R}$, and we ensure this by including a stuttering action sta that does not change the state, that is, $(s, sta, s) \in \mathcal{R}$; we restrict actions to be deterministic, that is, if $(s, a, s') \in \mathcal{R}$ and $(s, a, s'') \in \mathcal{R}$, then $s' = s''$; since actions are deterministic, sometimes we denote state s' as $s\langle a \rangle$ for which it holds that $(s, a, s\langle a \rangle) \in \mathcal{R}$. For convenience, we use $Ac(s) = \{a \mid \exists s' \in S : (s, a, s') \in \mathcal{R}\}$ to denote the available actions in state s.*
- *$s_0 \in S$ denotes the initial state.*

Now we define the language we use. The language $\mathcal{L}_{\mathrm{KA}}$, propositional logic extended with knowledge and action modalities, is generated by the following grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid K_i\varphi \mid \langle a \rangle \varphi \quad (i \in Agt, a \in Act)$$

The semantics of $\mathcal{L}_{\mathrm{KA}}$ are defined with respect to the satisfaction relation $\models$. Given a Kripke structure $\mathcal{T}$ and a state s in $\mathcal{T}$, a formula φ of the language can be evaluated as follows:

- $\mathcal{T}, s \models p$ iff $p \in \pi(s)$;
- $\mathcal{T}, s \models \neg\varphi$ iff $\mathcal{T}, s \not\models \varphi$;
- $\mathcal{T}, s \models \varphi_1 \vee \varphi_2$ iff $\mathcal{T}, s \models \varphi_1$ or $\mathcal{T}, s \models \varphi_2$;
- $\mathcal{T}, s \models K_i\varphi$ iff for all t such that $s\mathcal{K}(i)t$, $\mathcal{T}, t \models \varphi$;
- $\mathcal{T}, s \models \langle a \rangle\varphi$ iff there exists s' such that $(s, a, s') \in \mathcal{R}$ and $\mathcal{T}, s' \models \varphi$;

Other classical logic connectives (e.g.,"$\wedge$", "$\rightarrow$") are assumed to be defined as abbreviations by using $\neg$ and $\vee$ in the conventional manner. As is standard, we write $\mathcal{T} \models \varphi$ if $\mathcal{T}, s \models \varphi$ for all $s \in S$, and $\models \varphi$ if $\mathcal{T} \models \varphi$ for all Kripke structures $\mathcal{T}$.

In this paper, in addition of the $\mathcal{K}$-relation being S5, we also place restrictions of *no-forgetting* and *no-learning* based on Moore's work [8] for the simplification of our framework. It is defined as follows: given a state s in S, if there exists s' such that $s\langle a \rangle\mathcal{K}(i)s'$ holds, then there is a s'' such that $s\mathcal{K}(i)s''$ and $s' = s''\langle a \rangle$ hold; if there exists s' and s'' such that $s\mathcal{K}(i)s'$ and $s'' = s'\langle a \rangle$ hold, then $s\langle a \rangle\mathcal{K}(i)s''$. Following this restriction, we have $\models K_i(\langle a \rangle\varphi) \leftrightarrow \langle a \rangle K_i\varphi$. The *no-forgetting* principle says that if after performing action a agent i considers a state s' possible, then before performing action a agent i already considered possible that action a would lead to this state. In other words, if an agent has knowledge about the effect of an action, he will not forget about it after performing the action. The *no-learning* principle says that all the possible states resulting from the performance of action a in agent i's possible states before action a are indeed his possible states after action a. In other words, the agent will not gain extra knowledge about the effect of an action after performing the action.

3 Value System and Rational Alternative

Agents in the system are assumed to have their own value systems and knowledge. Based on their value systems and incomplete knowledge about the state, agents choose their rational alternatives for the next action they will perform.

3.1 Value System

Given several (possibly opportunistic) actions available to an agent, it is up to the agent's decision to perform opportunistic behavior. Basic decision theory applied to intelligent agents relies on three things: agents know what actions they can carry out, the effects of each action and agents' preference over the effects [10]. In this paper, the effects of each action are expressed by our logical language, and we will specify agents' abilities and preferences in this section. It is worth noting that we only study a single action being opportunistic in this paper, so we will apply basic decision theory for one-shot (one-time) decision problems, which concern the situations where a decision is experienced only once.

One important feature of opportunism is that it promotes agents' own value but demotes others' value. In this section we want to specify agents' value system, as it is the standard of agents' consideration about the performance of opportunistic behavior. A value can be seen as an abstract standard according to which agents have their preferences over states. For instance, if we have a value denoting *equality*, we prefer the states where equal sharing or equal rewarding hold. Related work about values can be found in [9] and [12].

Because of the abstract feature of a value, it is usually interpreted in more detail as a state property, which is represented as a $\mathcal{L}_{\mathrm{KA}}$ formula. The most basic value we can construct is simply a proposition p, which represents the value of achieving p. More complex values can be interpreted such as of the form $\langle a\rangle\varphi \wedge \langle a'\rangle\neg\varphi$, which represents the value that there is an option in the future to either achieve φ or $\neg\varphi$. Such a value corresponds to *freedom of choice*. A formula of a value can also be in the form of $K\varphi$, meaning that it is valuable to *achieve knowledge*. In this paper we denote values with v, and it is important to remember that v is an element from the language $\mathcal{L}_{\mathrm{KA}}$. However, not every formula from $\mathcal{L}_{\mathrm{KA}}$ can be intuitively classified as a value.

We argue that agents can always compare any two values, as we can consider two equivalent values as one value. In other words, every element in the set of values is comparable to each other and none of them is logically equivalent to each other. Therefore, we define a value system as a strict total order over a set of values, representing the degree of importance of something, which are inspired by the goal structure in [1] and [3].

Definition 3.1 (Value System). *A value system* $V = (\mathrm{Val}, \prec)$ *is a tuple consisting of a finite set* $\mathrm{Val} = \{v, ..., v'\} \subseteq \mathcal{L}_{\mathrm{KA}}$ *of values together with a strict total ordering* $\prec$ *over* Val. *When* $v \prec v'$, *we say that value* v' *is more important than value* v.

We also use a natural number indexing notation to extract the value of a value system, so if V gives rise to the ordering $v \prec v' \prec \dots$ then $V[0] = v$, $V[1] = v'$, and so on. Since a value is interpreted as a $\mathcal{L}_{\mathrm{KA}}$ formula and agents should be aware of the state property change for their value change, value promotion and demotion along a state transition can be defined as follows:

Definition 3.2 (Value Promotion and Demotion). *Given a value* v *and an action* a, *we define the following shorthand formulas:*

$$\mathrm{promoted}(v, a) := \neg v \wedge \langle a\rangle v$$
$$\mathrm{demoted}(v, a) := v \wedge \langle a\rangle\neg v$$

We say that a value v *is promoted along the state transition* (s, a, s') *if and only if* $s \models \mathrm{promoted}(v, a)$, *and we say that* v *is demoted along this transition if and only if* $s \models \mathrm{demoted}(v, a)$.

An agent's value v gets promoted along the state transition (s, a, s') if and only if v doesn't hold in state s and holds in state s'; an agent's value v gets demoted along the state transition (s, a, s') if and only if v holds in state s and doesn't hold in state s'. Note that in principle an agent is not always aware that his or her value gets demoted or promoted, i.e. it might be the case where $s \models \mathrm{promoted}(v, a)$ but agent i does not know this, i.e. $s \models \neg(K_i\,\mathrm{promoted}(v, a))$.

Now we can define a multi-agent system as a Kripke structure together with agents' value systems, representing their basis of practical reasoning. We also assume that value systems are common knowledge in the system to simplify the model. Formally, a multi-agent system $\mathcal{M}$ is an $(n+1)$-tuple: $\mathcal{M} = (\mathcal{T}, V_1, ..., V_n)$, where $\mathcal{T}$ is a Kripke structure, and for each agent i in $\mathcal{T}$, V_i is a value system.

We now define agents' preferences over two states in terms of values, which will be used for modelling the effect of opportunism. We first define a function $\mathrm{highest(i,s,s')}$ that maps a value system and two different states to the most preferred value that changes when going from state s to s' from the perspective of agent i. In other words, it returns the value that changes which the agent most cares about, i.e. the most important change between these states for the agent.

Definition 3.3 (Highest Value). *Given a multi-agent system $\mathcal{M}$, an agent i and two states s and s', function* $\mathrm{highest} : Agt \times S \times S \rightarrow \mathrm{Val}$ *is defined as follows:*

$$\mathrm{highest(i,s,s')}_{\mathcal{M}} := V_i[min\{j \mid \forall k > j : \mathcal{M}, s \models V_i[k] \Leftrightarrow \mathcal{M}, s' \models V_i[k]\}]$$

We write $\mathrm{highest(i,s,s')}$ *for short if $\mathcal{M}$ is clear from context.*

Note that if no values change between s and s', we have that $\mathrm{highest(i,s,s')} = V_i[0]$, i.e. the function returns the agents least preferred value. Moreover, it is not hard to see that $\mathrm{highest(i,s,s')} = \mathrm{highest(i,s',s)}$, meaning that the function is symmetric for the two state arguments.

With this function we can easily define agents' preference over two states. We use a binary relation "$\precsim$" over states to represent agents' preferences.

Definition 3.4 (State Preferences). *Given a multi-agent system $\mathcal{M}$, an agent i and two states s and s', agent i weakly prefers state s' to state s, denoted as $s \precsim_i^{\mathcal{M}} s'$, iff*

$$\mathcal{M}, s \models \mathrm{highest(i,s,s')} \Rightarrow \mathcal{M}, s' \models \mathrm{highest(i,s,s')}$$

We write $s \precsim_i s'$ for short if $\mathcal{M}$ is clear from context. Moreover, we write $S \precsim_i S'$ for sets of states S and S' whenever $\forall s \in S, \forall s' \in S' : s \precsim s'$.

As is standard, we also define $s \sim_i s'$ to mean $s \precsim_i s'$ and $s' \precsim_i s$, and $s \prec_i s'$ to mean $s \precsim_i s'$ and $s \not\succsim_i s'$. The intuitive meaning of the definition of $s \precsim_i s'$ is that agent i weakly prefers state s' to s if and only if the agent's most important value does not get demoted (either stays the same or gets promoted). In other words, agent i weakly prefers state s' to s: if $\mathrm{highest(i,s,s')}$ holds in state s, then it must also hold in state s', and if $\mathrm{highest(i,s,s')}$ does not hold in state s, then it does matter whether it holds in state s' or not. Clearly there is a correspondence between state preferences and promotion or demotion of values, which we can make formal with the following proposition.

Proposition 3.1. *Given a model $\mathcal{M}$ with agent i, state s and available action a in s. Let $v^* = \mathrm{highest(i, s, s\langle a\rangle)}$. We have:*

$$s \prec_i s\langle a\rangle \Leftrightarrow \mathcal{M}, s \models \mathrm{promoted}(v^*, a)$$
$$s \succ_i s\langle a\rangle \Leftrightarrow \mathcal{M}, s \models \mathrm{demoted}(v^*, a)$$
$$s \sim_i s\langle a\rangle \Leftrightarrow \mathcal{M}, s \models \neg(\mathrm{demoted}(v^*, a) \vee \mathrm{promoted}(v^*, a))$$

Proof. Firstly we prove the third one. We define $s \sim_i s\langle a\rangle$ to mean $s \precsim_i s\langle a\rangle$ and $s\langle a\rangle \precsim_i s$. $s \precsim_i s\langle a\rangle$ means that value v^* doesn't get demoted when going from s to $s\langle a\rangle$, and $s\langle a\rangle \precsim_i s$ means that value v^* doesn't get demoted when going from $s\langle a\rangle$ to s. Hence, value v^* doesn't get promoted or demoted (stays the same) by action a. Secondly we prove the first one. We define $s \prec_i s\langle a\rangle$ to mean $s \precsim_i s\langle a\rangle$ and $s \not\succ_i s\langle a\rangle$. $s \precsim_i s\langle a\rangle$ means that value v^* doesn't get demoted when going from s to $s\langle a\rangle$, and $s \not\succ_i s'$ means that either value v^* gets promoted or demoted by action a. Hence, value v^* gets promoted by action a. We can prove the second one in a similar way.

Additionally, apart from the fact that $s \prec_i s\langle a\rangle$ implies that the highest changed value gets promoted, we also have that no other value which is more preferred gets demoted or promoted. We have the result that the $\precsim_i$ relation obeys the standard properties we expect from a preference relation.

Proposition 3.2 (Properties of State Preferences). *Given an agent i, his preferences over states "$\precsim_i$" are*

- *Reflexive: $\forall s \in S : s \precsim_i s$;*
- *Transitive: $\forall s, s', s'' \in S$: if $s \precsim_i s'$ and $s' \precsim_i s''$, then $s \precsim_i s''$.*

Proof. The proof follows Definition 3.4 directly. In order to prove $\precsim_i$ is reflexive, we have to prove that for any arbitrary state s we have $s \precsim_i s$. From Definition 3.3 and Definition 3.4 we know $\mathrm{highest(i, s, s')} = V_i[0]$ when $s = s'$, and for any arbitrary state s we always have $\mathcal{M}, s \models V_i[0]$ implies $\mathcal{M}, s \models V_i[0]$. Therefore, $s \precsim_i s$ and we can conclude that $\precsim_i$ is reflexive.

In order to prove transitivity, we have to prove $\mathcal{M}, s \models v^*$ implies $\mathcal{M}, s'' \models v^*$, where $v^* = \mathrm{highest(i, s, s'')}$. It can be the case where v^* stays the same in state s and s'' or the case where $\mathcal{M}, s \models \neg v^*$ and $\mathcal{M}, s'' \models \neg v^*$. For the first case, when $s \sim s'$ and $s' \sim s''$, meaning that all the values stay the same when going from s to s' and from s' to s'', it is also the case when going from s to s''. We now consider the case where $\mathcal{M}, s \models \neg v^*$ and $\mathcal{M}, s'' \models \neg v^*$. Firstly, we denote $\mathrm{highest(i, s, s')}$ as u^* and $\mathrm{highest(i, s', s'')}$ as w^*. It can either be that $u^* \sim_i w^*$, $u^* \prec_i w^*$ or $u^* \succ_i w^*$. If $u^* \sim_i w^*$, we can conclude that $u^* \sim_i w^* \sim_i v^*$, hence the implication holds. We now distinguish between the cases where $u^* \prec_i w^*$ or $u^* \succ_i w^*$.

- If $u^* \prec_i w^*$, we know that w^* is the highest value that changes and gets promoted when going from s' to s'', but stays the same between s and s'. Hence, we can conclude that $\mathcal{M}, s \models \neg w^*$ and $\mathcal{M}, s'' \models w^*$, and that $w^* = v^*$ (i.e., w^* is the highest value that changes between s and s''). Hence we have $\mathcal{M}, s \models v^*$ implies $\mathcal{M}, s'' \models v^*$.

- If $u^* \succ_i w^*$, we know that u^* is the highest value that changes and gets promoted when going from s to s', but stays the same between s' and s''. Hence, we can conclude that $\mathcal{M}, s \models \neg u^*$ and $\mathcal{M}, s'' \models u^*$, and that $u^* = v^*$ (i.e. v^* is the highest value that changes between s and s''). Hence we have $\mathcal{M}, s \models v^*$ implies $\mathcal{M}, s'' \models v^*$.

In our system, we only look at the highest value that changes to deduce state preferences. Certainly, there are other ways of deriving these preferences from a value system. Instead of only considering the highest value change in the state transition, it is also possible to take into account all the value changes in the state transition. For opportunism, what we want to stress is that opportunistic agents ignore (rather than consider less) other agents' interest, which has a lower index in the agent's value system. In order to align with this aspect, we use the highest value approach in this paper.

3.2 Rational Alternatives

Since we have already defined values and value systems as agents' standards for decision-making, we can start to apply decision theory to reason about agents' decision-making. Given a state in the system, there are several actions available to an agent, and he has to choose one in order to go to the next state. We can see the consideration here as a one-shot decision making. In decision theory, if agents only act for one step, a rational agent should choose an action with the highest (expected) utility without reference to the utility of other agents [10]. Within our framework, this means that a rational agent will always choose an rational alternative based on his value system.

Before choosing an action to perform, an agent must think about which actions are available to him. We have already seen that for a given state s, the set of available actions is $Ac(s)$. However, since an agent only has partial knowledge about the state, we argue that the actions that an agent knows to be available is only part of the actions that are physically available to him in a state. For example, an agent can call a person if he knows the person's phone number; without this knowledge, he is not able to do it, even though he is holding a phone. Recall that the set of states that agent i considers as being the actual state in state s is the set $\mathcal{K}(i, s)$. Given an agent's partial knowledge about a state as a precondition, he knows what actions he can perform in that state, which is the intersection of the sets of actions physically available in the states in this knowledge set.

Definition 3.5 (Subjectively Available Actions). *Given an agent i and a state s, agent i's subjectively available actions are the set:*

$$Ac(i, s) = \bigcap_{s' \in \mathcal{K}(i,s)} Ac(s').$$

Because a stuttering action sta is always included in $Ac(s)$ for any state s, we have that $sta \in Ac(i, s)$ for any agent i. When only sta is in $Ac(i, s)$, we say that the agent cannot do anything because of his limited knowledge. Obviously an agent's subjectively available actions is always part of his physically available actions ($Ac(i,s) \subseteq Ac(s)$). Based on rationality assumptions, he will choose an action based on his partial knowledge of the current state and the next state. Given a state s and an action a, an agent considers the next possible states as the set $\mathcal{K}(i, s\langle a\rangle)$. For another action a', the set of possible states is $\mathcal{K}(i, s\langle a'\rangle)$. The question now becomes: How do we compare these two possible set of states? Clearly, when we have $\mathcal{K}(i, s\langle a\rangle) \precsim_i \mathcal{K}(i, s\langle a'\rangle)$, meaning that all alternatives of performing action a' are at least as desirable as all alternatives of choosing action a, it is always better to choose action a'. However, in some cases it might be that some alternatives of action a are better than some alternatives of action a' and vice-versa. In this case, an agent cannot decisively conclude which of the actions is optimal. This approach has natural ties to game theory in the context of (non-)dominated strategies [5]. This leads us to the following definition:

Definition 3.6 (Rational Alternatives). *Given a state s, an agent i and two actions $a, a' \in Ac(i,s)$, we say that action a is* dominated *by action a' for agent i in state s iff $\mathcal{K}(i, s\langle a\rangle) \precsim_i \mathcal{K}(i, s\langle a'\rangle)$. The set of* rational alternatives *for agent i in state s is given by the function $a_i^* : S \to 2^{Act}$, which is defined as follows:*

$$a_i^*(s) = \{a \in Ac(i,s) \mid \neg\exists a' \in Ac(i,s) : a \neq a' \textit{ and } a' \textit{ dominates } a \textit{ for agent } i \textit{ in state } s\}.$$

The set $a_i^*(s)$ are all the actions for agent i in state s which are available to him and are not dominated by another action which is available to him. In other words, it contains all the actions which are rational alternatives for agent i. Since it is always the case that $Ac(i,s)$ is non-empty because of the stuttering action sta, and since it is always the case that there is one action which is non-dominated by another action and $Ac(i,s)$ is finite, we conclude that $a_i^*(s)$ is non-empty. We can see that the actions that are available to an agent not only depend on the physical state, but also depend on his knowledge about the state. The more he knows, the better he can judge what his rational alternative is. In other words, agents try to make a best choice based on their value systems and incomplete knowledge about the state. The following proposition shows how agents remove actions with our approach.

Proposition 3.3. *Given a state s, an agent i and two actions $a, a' \in Ac(i,s)$, action a is dominated by action a' iff*

$$\neg\exists s', s'' \in \mathcal{K}(i,s) : s'\langle a\rangle \succ s''\langle a'\rangle.$$

Proof. $\exists s', s'' \in \mathcal{K}(i,s) : s'\langle a\rangle \succ s''\langle a'\rangle$ is equivalent to $\mathcal{K}(i, s\langle a\rangle) \not\precsim \mathcal{K}(i, s\langle a'\rangle)$, because $s'\langle a\rangle \in \mathcal{K}(i, s\langle a\rangle)$ and $s''\langle a'\rangle \in \mathcal{K}(i, s\langle a'\rangle)$. And $\mathcal{K}(i, s\langle a\rangle) \not\precsim \mathcal{K}(i, s\langle a'\rangle)$ is equivalent to the fact that action a is non-dominated by action a'.

From this proposition we can see that agents remove all the options (actions) that are always bad to do, and there is no possibility to be better off by choosing a dominated action. The following proposition connects Definition 3.6 with state preferences.

Proposition 3.4. *Given a multi-agent system $\mathcal{M}$, a state s and an agent i,*

$$sta \notin a^*(s) \Rightarrow \forall a \in a^*(s) : s \precsim_i s\langle a \rangle.$$

Proof. We prove it by contradiction. If there exists an action $a \in a^*(s)$ such that agent i's value will get demoted by performing it, it will be dominated by the stuttering action sta, which can always keep agent i's values neutral, and sta might be in $a^*(s)$. Contradiction!

If the stuttering action sta is not in the set of rational alternatives for agent i, meaning that it is dominated by the actions in the set of rational alternatives, agent i can always at least keep his value neutral by performing any action in his rational alternatives. We will illustrate the above definitions and our approach through the following example.

Example 1. Figure 1 shows a transition system $\mathcal{M}$ for agent i. State s and s' are agent i's epistemic alternatives, that is, $\mathcal{K}(i, s) = \{s, s'\}$. Now consider the actions that are physically available and subjectively available to agent i. $Ac_i(s) = \{a_1, a_2, a_3, sta\}$, $Ac_i(s') = \{a_1, a_2, sta\}$. Because $Ac(i, s) = Ac_i(s) \cap Ac_i(s')$, agent i knows that only sta, a_1 and a_2 are available to him in state s .

Next we talk about agent i's rational alternatives in state s. Given agent i's value system $V_i = (u \prec v \prec w)$, and the following valuation: u, $\neg v$ and $\neg w$ hold in $\mathcal{K}(i, s)$, $\neg u$, $\neg v$ and w hold in $\mathcal{K}(i, s\langle a_1 \rangle)$, and u, v and $\neg w$ hold in $\mathcal{K}(i, s\langle a_2 \rangle)$, we then have the following state preferences: $\mathcal{K}(i, s) \prec \mathcal{K}(i, s\langle a_1 \rangle)$, $\mathcal{K}(i, s) \prec \mathcal{K}(i, s\langle a_2 \rangle)$ and $\mathcal{K}(i, s\langle a_2 \rangle) \prec \mathcal{K}(i, s\langle a_1 \rangle)$, meaning that action a_2 and the stuttering action sta are dominated by action a_1. Thus, we have $a_i^*(s) = \{a_1\}$.

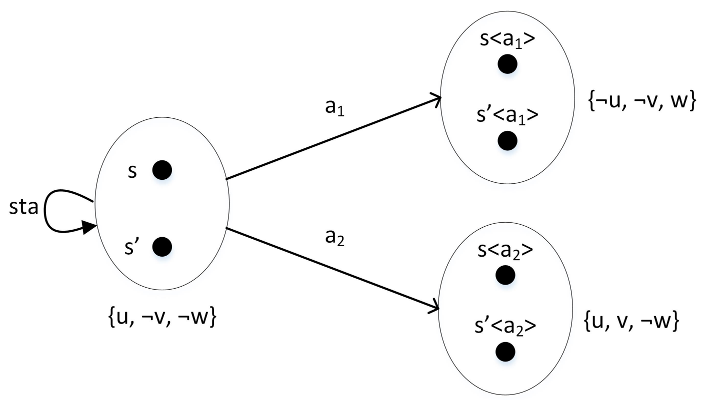

Fig. 1. A transition system $\mathcal{M}$ for agent i

4 Defining Opportunism

Before reasoning about opportunistic propensity, we should first formally know what opportunism actually is. Opportunism is a social behavior that takes advantage of relevant knowledge asymmetry and results in promoting one's own value and demoting others' value [6]. It means that it is performed with the precondition of relevant knowledge asymmetry and the effect of promoting agents' own value and demoting others' value. Firstly, knowledge asymmetry is defined as follows.

Definition 4.1 (Knowledge Asymmetry). *Given two agents i and j, and a $\mathcal{L}_{\text{KA}}$ formula ϕ, knowledge asymmetry about ϕ between agent i and j is the abbreviation:*

$$\text{KnowAsym}(i, j, \phi) := K_i\phi \wedge \neg K_j\phi \wedge K_i(\neg K_j\phi)$$

It holds in a state where agent i knows ϕ while agent j does not know ϕ and this is also known by agent i. It can be the other way around for agent i and agent j. But we limit the definition to one case and omit the opposite case for simplicity. Now we can define opportunism.

Definition 4.2 (Opportunism). *Given a multi-agent system $\mathcal{M}$, a state s and two agents i and j, the assertion* $\text{Opportunism}(i, j, a)$ *that action a performed by agent i is opportunistic behavior is defined as:*

$$\text{Opportunism}(i, j, a) := \text{KnowAsym}(i, j, \text{promoted}(v^*, a) \wedge \text{demoted}(w^*, a))$$

where $v^* = \text{highest}(\text{i}, \text{s}, \text{s}\langle\text{a}\rangle)$ *and* $w^* = \text{highest}(\text{j}, \text{s}, \text{s}\langle\text{a}\rangle)$.

This definition shows that if the precondition KnowAsym is satisfied in state s then the performance of action a will be opportunistic behavior. The asymmetric knowledge that agent i has is about the change of the truth value of v^* and w^* along the transition by action a, where v^* and w^* are the values that agent i and agent j most care about along the transition respectively. It follows that agent j is partially or completely not aware of it. Compared to the definition of opportunism in [6], Definition 4.2 focuses on the opportunistic propensity of an agent in a state, in the sense that the precondition of performing opportunistic behavior is modeled in an explicit way. As is stressed in [6], opportunistic behavior is performed by intent rather than by accident. In this paper, instead of explicitly modeling intention, we interpret it from agents' rationality that they always intentionally promote their own values. We can derive three propositions from the definition, which are useful in our next section.

Proposition 4.1 (Value Promotion and Demotion). *Given a multi-agent system $\mathcal{M}$ and an opportunistic behavior a performed by agent i to agent j in state s, action a will promote agent i's value but demote agent j's value, which can be formalized as*

$$\mathcal{M}, s \models \text{Opportunism}(i, j, a) \quad \Rightarrow \quad s \prec_i s\langle a\rangle \textit{ and } s \succ_j s\langle a\rangle$$

Proof. From $\mathcal{M}, s \models \text{Opportunism}(i, j, a)$ we have: $\mathcal{M}, s \models K_i(\text{promoted}(v^*, a) \wedge \text{demoted}(w^*, a))$. And thus since all knowledge is true, we have that $\mathcal{M}, s \models \text{promoted}(v^*, a)$ and $\mathcal{M}, s \models \text{demoted}(w^*, a)$. Using the correspondence found in Proposition 3.1, we can conclude $s \prec_i s\langle a\rangle$ and $s \succ_j s\langle a\rangle$.

Proposition 4.2 (Different Value Systems). *Given a multi-agent system $\mathcal{M}$ and opportunistic behavior a performed by agent i to agent j in state s, agent i and agent j have different value systems, which can be formalized as*

$$\mathcal{M}, s \models \text{Opportunism}(i, j, a) \quad \Rightarrow \quad V_i \neq V_j$$

Proof. We prove it by contradiction. We denote $v^* = \text{highest(i, s, s}\langle\text{a}\rangle)$ and $w^* = \text{highest(j, s, s}\langle\text{a}\rangle)$, for which v^* and w^* are the property changes that agent i and agent j most care about in the state transition. If $V_i = V_j$, then $v^* = w^*$. However, because $\mathcal{M}, s \models K_i(\text{promoted}(v^*, i) \wedge \text{demoted}(w^*, j))$, and thus $\mathcal{M}, s \models K_i(\neg v^* \wedge w^*)$, and because knowledge is true, we have $\mathcal{M}, s \models \neg v^* \wedge w^*$. But, since $v^* = w^*$, we have $\mathcal{M}, s \models \neg v^* \wedge v^*$. Contradiction!

From this proposition we can see that agent i and agent j care about different things based on their value systems about the transition.

Proposition 4.3 (Inclusion). *Given a multi-agent system $\mathcal{M}$ and opportunistic behavior a performed by agent i to agent j in state s, agent j's knowledge set in state s is not a subset of agent i's and action a is available in agent i's knowledge set:*

$$\mathcal{M}, s \models \text{Opportunism}(i, j, a) \quad \Rightarrow \mathcal{K}(j, s) \not\subseteq \mathcal{K}(i, s) \text{ and } a \in Ac(i, s)$$

Proof. We can prove it by contradiction. Knowledge set is the set of states that an agent considers as possible in a given actual state. $\forall t \in \mathcal{K}(i, s)$, agent i considers state t as a possible state where he is residing. The same with $\mathcal{K}(j, s)$ for agent j. If $\mathcal{K}(j, s) \not\subseteq \mathcal{K}(i, s)$ is false, we have $\mathcal{K}(j, s) \subseteq \mathcal{K}(i, s)$ holds, which means that agent j knows more than or exactly the same as agent i. However, Definition 4.2 tells that agent i knows more about the transition by action a than agent j. So $\mathcal{K}(j, s) \subseteq \mathcal{K}(i, s)$ is false, meaning that $\mathcal{K}(j, s) \not\subseteq \mathcal{K}(i, s)$ holds. Further, because from $\mathcal{M}, s \models \text{Opportunism}(i, j, a)$ we have $\mathcal{M}, s \models K_i(\langle a\rangle v^* \wedge \langle a\rangle \neg w^*)$, by the semantics of $\langle a\rangle v^*$ and $\langle a\rangle \neg w^*$, for all $t \in \mathcal{K}(i, s)$ there exists $(t, a, s') \in R$. Thus, we have $a \in Ac(i, s)$.

These three propositions are three properties that we can derive based on Definition 4.2. The first one shows that opportunistic behavior results in value opposition for the agents involved; the second one tells that the two agents involved in the relationship evaluate the transition based on different value systems; the third one indicates the asymmetric knowledge that agent i has for behaving opportunistically. We will illustrate the above definitions through the example mentioned at the beginning of the paper.

Example 2. Figure 2 shows the example of selling a broken cup: The action selling a cup is denoted as *sell* and we use two value systems V_s and V_b for the seller and the buyer respectively. State s_1 is the seller's epistemic alternative, while state s_1 and s_2 are the buyer's epistemic alternatives. We also use a dash line circle to represent the buyer's knowledge $\mathcal{K}(b, s_1)$ (not the seller's). In this example, $\mathcal{K}(s, s_1) \subset \mathcal{K}(b, s_1)$. Moreover, $hm = \mathrm{highest}(\mathrm{s}, \mathrm{s}_1, \mathrm{s}_1\langle \mathrm{sell}\rangle)$, $\neg hb = \mathrm{highest}(\mathrm{b}, \mathrm{s}_1, \mathrm{s}_1\langle \mathrm{sell}\rangle)$, meaning that the seller only cares about if he gets money from the transition, while the buyer only cares about if he has a broken cup from the transition. We also have $\mathcal{M}, s_1 \models K_s(\mathrm{promoted}(hm, sell) \wedge \mathrm{demoted}(\neg hb, sell))$, meaning that the seller knows the transition will promote his own value while demote the buyer's value in state s_1. For the buyer, action *sell* is available in both state s_1 and s_2. However, hb doesn't hold in both s_1 and s_2, so he doesn't know if he has a broken cup or not. Therefore, there is knowledge asymmetry between the seller and the buyer about the value changes from s_1 to $s_1\langle sell\rangle$. Action *sell* is potentially opportunistic behavior in state s_1.

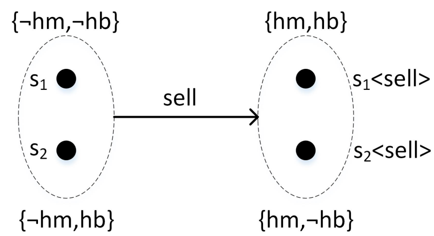

Fig. 2. Selling a broken cup

5 Reasoning about Opportunistic Propensity

In this section, we will characterize the contexts where agents will perform opportunistic behavior and where opportunism is impossible to happen.

5.1 Having Opportunism

Agents will perform opportunistic behavior when they have the ability and the desire of doing it. The ability of performing opportunistic behavior can be interpreted by its precondition: it can be performed whenever its precondition is fulfilled. Agents have desire to perform opportunistic behavior whenever it is a rational alternative. There are also relations between agents' ability and desire of performing an action. As rational agents, firstly we think about what actions we can perform given the limited knowledge we have about the state, and secondly we choose the action that may maximize our utilities based on our partial knowledge. This practical reasoning in decision theory can also be applied to reasoning about opportunistic propensity. Given the asymmetric knowledge an agent has, there are several (possibly opportunistic) actions available to him, and he may choose to perform the action which is a rational alternative to him, regardless of the result for the other agents. Based on this understanding, we have the following theorem, which implies agents' opportunistic propensity:

Theorem 1. *Given a multi-agent system $\mathcal{M}$, a state s, two agents i and j and opportunistic behavior a, opportunistic behavior is a rational alternative for agent i in state s:*

$$\exists a \in a_i^*(s) : \mathcal{M}, s \models \text{Opportunism}(i, j, a)$$

iff

1. $\forall t \in \mathcal{K}(i, s) : \mathcal{M}, t \models \text{promoted}(v^*, a) \wedge \text{demoted}(w^*, a)$, $\exists t \in \mathcal{K}(j, s) : \mathcal{M}, t \models \neg(\text{promoted}(v^*, a) \wedge \text{demoted}(w^*, a))$, *where* $v^* = \mathrm{highest(i, s, s\langle a\rangle)}$ *and* $w^* = \mathrm{highest(j, s, s\langle a\rangle)}$*;*
2. $s \prec_i s\langle a\rangle$ *and* $s \succ_j s\langle a\rangle$*;*
3. $\neg\exists a' \in Ac(i, s) : a \neq a'$ *and* a' *dominates* a*.*

Proof. **Forwards:** If action a is opportunistic behavior, we can immediately have statement 1 by the definition of Knowledge Set. Because action a is in agent i's rational alternatives in state s ($a \in a_i^*(s)$), by Definition 3.6, action a is not dominated by any action in $Ac(i, s)$. Also because action a is opportunistic, by Proposition 4.1 it results in promoting agent i's value but demoting agent j's value ($s \prec_i s\langle a\rangle$ and $s \succ_j s\langle a\rangle$). **Backwards:** Statement 1 means that there is knowledge asymmetry between agent i and agent j about the formula $\text{promoted}(v^*, a) \wedge \text{demoted}(w^*, a)$. From this we can see the knowledge asymmetry is the precondition of action a. If this precondition is satisfied, agent i can perform action a. Moreover, by statement 2, because action a promotes agent i's value but demotes agent j's value, we can conclude that action a is opportunistic behavior. By statement 3, because action a is not dominated by any action in $Ac(i, s)$, it is a rational alternative for agent i in state s to perform action a.

Given an opportunistic behavior a, in order to predict its performance, we should first check the asymmetric knowledge that agent i has for enabling its performance. Based on agent i's and agent j's value systems, we also check if it is not dominated by any actions in $Ac(i, s)$ and its performance can promote agent i's value but demote agent j's value. It is important to stress that Theorem 1 never states that agents will for sure perform opportunistic behavior if the three statements are satisfied. Instead, it shows opportunism is likely to happen because it is in agents' rational alternatives.

5.2 Not Having Opportunism

We need much information about the system as Theorem 1 states to predict opportunism, and it might be difficult to achieve all of them. Fortunately, sometimes it is already enough to know that opportunism is impossible to occur.

An example might be detecting opportunism: if we already know in which context agents cannot perform opportunistic behavior, there is no need to set up any monitoring mechanisms for opportunism in those contexts. The following propositions characterize them:

Proposition 5.1. *Given a multi-agent system $\mathcal{M}$, a state s, two agents i and j and an action a,*

$$\mathcal{K}(i,s) = \mathcal{K}(j,s) \Rightarrow \mathcal{M}, s \models \neg Opportunism(i,j,a).$$

Proof. When $\mathcal{K}(i,s) = \mathcal{K}(j,s)$ holds, which means that both agent i and agent j have the same knowledge. In this context, Statement 1 in Theorem 1 is not satisfied, so action a is not opportunistic behavior.

Proposition 5.2. *Given a multi-agent system $\mathcal{M}$, a state s, two agents i and j and an action a,*

$$V_i = V_j \Rightarrow \mathcal{M}, s \models \neg Opportunism(i,j,a).$$

Proof. If $V_i = V_j$ holds, which means that both agent i and agent j have the same value system, the values of both agents don't go opposite, that is, Statement 2 in Theorem 1 is not satisfied. So action a is not opportunistic behavior.

In this section, we specified the situation where agents will perform opportunistic behavior and characterized the contexts where opportunism is impossible to happen. This information is essential not only for the system designers to identify opportunistic propensity, but also for an agent to decide whether to participate in the system given his knowledge and value system, as his behavior might be regarded as opportunistic. Moreover, our approach can be used in practice. For instance, in the electronic market place, only the seller knows that the product is not good for the buyer before he ships it, and he can earn more money if he still claims that the product is good. In this context the seller can and wants to perform opportunistic behavior, selling the product, to the buyer according to Theorem 1. Monitoring and constraint mechansim should be put there in order to demotivated such a behavior. However, if we can ensure that both the seller and the buyer are aware of the quality of the product before the seller ships it, it is impossible for him to get benefits from the buyer.

6 Conclusion and Future Work

The investigation about opportunism is still new in the area of multi-agent system. We ultimately aim at designing a constraint mechanism to eliminate such selfish behavior in the system. In order to avoid over-assuming the performance of opportunism so that monitoring and constraint mechanism can be put in place, we need to know in which context agents will or are likely to perform opportunistic behavior. In this paper, we argue that agents will behave opportunistically when they have the ability and the desire of doing it. With this idea, we developed a framework of multi-agent systems to reason about agents' opportunistic propensity without considering normative issues. Agents in the system were assumed to have their own value systems. Based on their value systems and incomplete knowledge about the state, agents choose one of their rational alternatives, which might be opportunistic behavior. With our framework and our definition of opportunism, we characterized the situation where agents will perform opportunistic behavior and the contexts where opportunism is impossible to occur. Certainly there are multiple ways to extend our work. One interesting way is to enrich our formalization of value system over different sets of values, and the enrichment might lead to a different notion of the compatibility of value systems and different results about opportunistic propensity. Another way is to consider normative issues in our framework in addition to the ability and the desire of being opportunistic.

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